Evidentiary Analysis of Traffic Accidents

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1. Fundamental Quantities which form the basis for all the derived mechanical variables are:

   1. length (e.g., feet, miles, etc.) (distance)
   2. mass (slugs not pounds) = 32.2 lbs. MASS ≠ WEIGHT
   3. time (e.g., seconds, hours, etc.) → WEIGHT = M x g

MASS = ACCELERATION DUE TO GRAVITY

Except for mass we have experience using the names and units of the fundamental quantities. Notice that if one is to understand the meaning of a numerical value for length or time then the unit or label must accompany the number.

e.g. 1. How far did you travel today? To answer intelligently you must give the unit along with the number. That is, an answer of 5 is nonsense while an answer of 5 miles means something to the questioner.

2. How long have you been traveling? Again to make someone understand your reply you must give a label or unit along with the number. An answer of $3\frac{1}{2}$ does not make sense. Rather, you need to say $3\frac{1}{2}$ days (or hours or minutes, etc.)

2. Derived mechanical quantities are always combinations of the fundamental quantities. In what follows, we shall define the derived mechanical quantities, provide the required combination of the fundamental units and relate the physical definitions to your own everyday usage of the terms.

AVERAGE → SPEED = DISTANCE TRAVELED / TIME IT TAKES $V = \frac{d}{t}$ 15 mph = $\frac{15}{30}$ = $\frac{1}{2}$

I. a) Speed or Velocity In our everyday language we use speed and velocity interchangeably. There is, however, a slight scientific difference in the two. Specifically, velocity is the term which is used when a direction is associated with a numerical speed. Coincidentally, one of our best notions of speed comes from the automobile. The speedometer tells us how fast we are going - say 28 miles per hour. The word per is used to indicate the mathematical operation of division. Therefore,

$$28 \text{ miles per hour} = 28 \frac{\text{miles}}{\text{hour}}$$

once you give speed a direction you have velocity

Notice the unit associated with speed is a particular combination of length and time, namely, length divided by time. Symbolically, we use letters to represent the physical quantities as follows:

$$(\text{speed}) \quad v = \frac{d}{t} \quad \frac{(\text{distance})}{(\text{time})}$$

d and t are fundamental quantities and v is a derived quantity whose unit is a special combination of the fundamental quantities. Note that as one travels a distance of 88 feet in two seconds then the average speed is

$$v = \frac{88}{2} \frac{ft}{sec} = 44 \frac{ft}{sec}$$

or if d= 52 miles and t = 4 hrs then

$$v = \frac{52}{4} = 13 \frac{\text{miles}}{\text{hour}}$$

In the last example you can imagine the individual traveling as follows :

$25 \frac{mi}{hr}$ for the 1st hour

Stopped ($v = 0$) for two hours

and, $27 \frac{mi}{hr}$ for the 4th hour

Obviously he has traveled

25+0+0+ 27 = 52 miles

in the 4 hours but at no time was he traveling at 13 miles/hour. This example illustrates that the formula

$$v = d/t$$

really gives only an average speed. When we associate a direction with a speed (e.g., 30 miles/hour south) then the physical quantity is called a velocity and the numerical value 30 miles/hour is termed the speed.

Often in the analysis of traffic accidents, the desired results include the speeds of the participating vehicles before evasive action was taken prior to a collision. As we shall see later these results are impossible to obtain unless the directions of the speeds before and after skids and collisions can be assessed from the physical evidence at the accident site. Therefore, one must know one part of the velocities (directions) before he can compute the other part (the magnitudes).

b. <u>Acceleration</u> - People who drive a car have a feel for the definition of acceleration. When the car speeds up we say it is accelerating. Furthermore, the quicker it speeds up then the greater is the acceleration. When the driver steps on the brake we describe the slowing down or speed reduction as deceleration. The physical or scientific definition agrees with the above intuitive notion of acceleration. Specifically, acceleration is define4 as follows:

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{the time period required to change the velocity}}$$

Using Symbols:

$$a = \frac{V_{final} - V_{initial}}{t}$$

Where V_{final} is the velocity at the end of the time interval and $V_{initial}$ is the velocity at the beginning of the time interval. Sometimes $V_{final} - V_{initial}$ is called ΔV or the change in velocity.

$$a = \frac{\Delta V}{t}$$

Δ = CHANGE

Alternately we can write

NB

$$a = \Delta V \times \frac{1}{t}$$

1 mile = 5280 ft = 1.466 ft/sec 60 mph = 88'/sec.

In this way we can determine the label or unit for acceleration. Let

$$V_{final} = 75 \text{ ft/sec}$$

$$V_{initial} = 25 \text{ ft/sec} \quad \text{and} \quad t = 5 \text{ sec}$$

$$\therefore \Delta V = V_{final} - V_{initial} = 75 - 25 = 50 \text{ ft/sec}$$

Then:

$$a = \Delta V \times \frac{1}{t} = 50 \frac{ft}{sec} \times \frac{1}{5 sec}$$

$$a = \frac{50}{5} \frac{ft}{sec\ sec}$$

$$a = 10 \frac{ft}{sec^2}$$

That is, the unit for acceleration is length divided by time squared. Again, this is a derive4 unit and coED-rised of % special combination of the fundamental units of length and time. If the final velocity is less than the initial velocity then the acceleration is negative. Consider me following example

$v_{final} = 10 \frac{ft}{sec}$; $v_{initial} = 30 \frac{ft}{sec}$; $t = 2$ sec

$$a = \frac{\Delta v}{t} = \frac{10 - 30}{2} \frac{ft}{sec^2}$$

$$a = -10 \frac{ft}{sec^2}$$

(Slowing down / [illegible])

The negative sign indicztes that the final velocity must be less than the initial velocity. Therefore, negative acceleration is just slowing down or deceleration.

There is a third accelerator (besides the gas pedal and the brake) in a car. It is the steering wheel because it allows one to change direction of the velocity. This change in velocity is a real acceleration even if tie speed remains a constant. You may have heard it referred to as centrifugal acceleration. We will return to it later.

Isaac Newton

c. Force - In our everyday experience the use of forces are comm[illegible] [illegible]e speak of forces in terms of pushes or pulls, [illegible] forcing a stuck door open, of pushing a stalled [illegible] exerting a force to move a heavy object (e.g., a piano). From our own experience we know that sometimes forces cause motion, sometimes they are used to slow something down, and sometimes no matter how hard we push no motion results. These notions and experiences which we all hsve were formulated in three physical laws by Newton. Old Isaac was able to represent all the physical consequences of forces in very simple mathematical laws. They are as follows:

3 laws

1. [illegible] an object; is in a state of uniform motion in [illegible] aight line (i.e., constant speed) or at rest it will remain that way unless it is acted on by external force.

What Newton noticed is that in order to ~ ~ u[illegible]e object to speed up or to cause an object to slow down a force has to act, that is, a force is required to change X object's speed.

[illegible] mass - harder to produce same acceleration w/ same force

force = mass × acceleration

2. The acceleration of an object (i.e., its change in velocity) is proportional to the net force applied to the object.

$$F \propto a$$

That is, Newton noticed that as one increased the net (or unbalanced) force acting on a body then its acceleration (or deceleration) increased. For example, you stop a car quicker (get greater deceleration) by pushing on the brake with a greater force. Newton realized that matter itself resisted the efforts of a force to cause acceleration. For example, it is easier to push a VW out of a ditch than to push a loaded pickup truck out of a ditch. That is, if you double the matter (or weight) you need twice the force to cause the same acceleration. Mathematically we write

Pounds

$$F = m \times a \;\; \frac{ft}{sec^2} \qquad m = \text{mass (slug)}$$

$$1\,lb = 1\,slug \times 1\,ft/sec^2$$

where the quantity of matter - or - the resistance to acceleration is called mass.

The unit of mass is called a slug. Therefore the unit of force can be derived as follows

$$F = m\,(slugs)\; a\left(\frac{ft}{sec^2}\right)$$

$$F \text{ has units of } \frac{slug \cdot ft}{sec^2}$$

$F_{net} = F - F_f$ (Force of Friction) $= 0$

We usually call this unit a pound. That is, the unit of force is a pound. We know that weight is given in pounds. Therefore, it follows that an object's weight represents a force. The force in this case is the force of attraction due to the gravitational attraction of all objects on the earth to the earth itself. The force of gravity (or weight) is a very special force and has the form

$$W = mg$$

where g is termed the acceleration due to gravity. Its value at the surface of the earth is

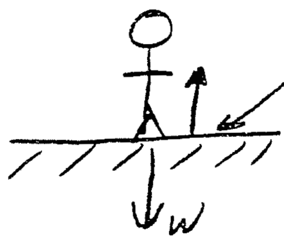
$$g = 32\; ft/sec^2$$

4-11 objects if allowed to fall in the absence of sir (therefore, no air friction to oppose the fall) will fall with *this* acceleration. Note that if you took your car to the noon it would weigh less because the gravitational force on the moon is less than on earth (i.e., the acceleration due to gravity on the moon is less than 32 ft/sec2) but the car still is physically the same in that it is composed of the same atoms and molecules as before. That is, the weight is <u>less</u> but the <u>mass</u> is the <u>same</u>.

Even though an object is not falling freely we still represent its weight by the formula:

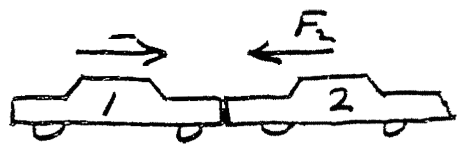
$$W = mg \quad \left(\text{e.g. } m = 5 \text{ slugs},\ g = 32 \text{ ft/sec}, \quad W = 5 \times 32 = 160 \text{ lbs} \right)$$

If the object is *not* falling then other forces are opposing the gravitational force, e.g.,



Here the floor pushes up and opposes gravity. Since there is no acceleration ($a = 0$), the force exerted by the floor is equal and opposite to W. They balance

3. Newton's 3rd law states that for every sction force *there* is an equal or **opposite** reaction force. Consider the collision between two cars. Car #1 exerts a force F_1 on car #2 and car # 2 exerts a force on car #1.



That is the forces are equal in magnitude (ssy 300 lbs. each) but directed in exactly opposite directions. mathematically we can **designate** directions to the right as positive and to the left as negative. Thus

$$(\text{action})\ F_1 = -F_2\ (\text{reaction})$$

Notice that F_1 and F_2 are on different bodies (here, car #2 and car #1, respectively). This law allows the derivation of the principle of conservation of <u>momentum</u> which is the single most important principle which is used in the analysis of auto collisions. We will introduce momentum after defining and discussing energy.

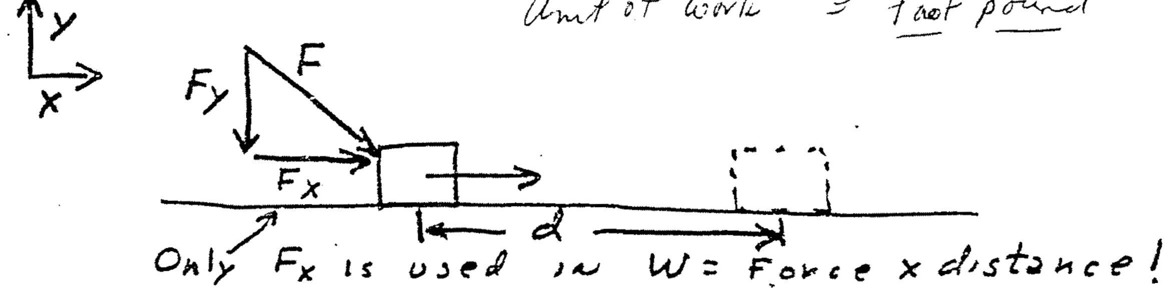
d. Work -- Again we all have an understanding of work. Of particular interest for the purposes of this workshop is mechanical work. The scientific definition turns out to be in accord with our own notions of physical work. In particular, consider the following examples of doing work:
a) turning a crank, b) pushing or pulling furniture around when rearranging a room, c) shoveling snow etc. In each case, a force is exerted through a distance. Exerting a force through a distance is exactly the scientific notion of work and we calculate it by multiplying the force times the distance.

Work = Force x distance

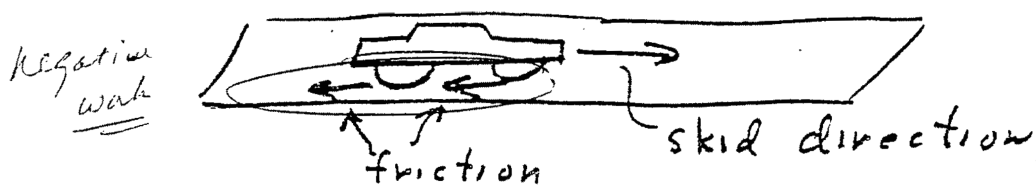
Using Symbols → $W = F \times d$

The unit is ft-lbs (just the unit of force times the unit of length). The only restriction on the scientific definition is that the force must act along the direction of movement. If the force and the direction of movement are not parallel then only the portion of the force which is parallel to the direction of motion appears in the formula.

Unit of work = foot pound



A very special force which must be considered in accident analysis is the frictional force (f) between the road surface and skidding rubber tires. Consider the figure below



Locked tires skid on the surface. Thus, the frictional force acts for the entire length of the skid. Since the skid direction is directly opposite to the direction of the frictional force the work done by the frictional force is negative

$$W = - f \times d$$

Negative work always represents the dissipation of motional energy. In this case the car slows down.

Friction = μ

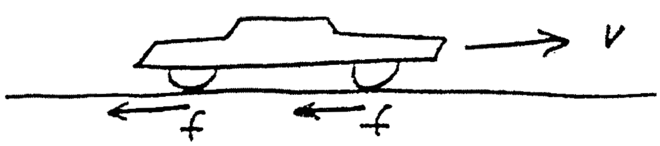
e. Frictional Force - In an emergency situation which calls for a driver to stop his automobile quickly, one instinctively pushes as hard and as fsat as possible on the brake pedal. The typical result is that the car will begin to skid because the wheels are locked. The driver is therefore making use of frictional forces between the road surface and the four locked tires to slow down the car.

The frictional force is familiar to all of us. We use it to our benefit not only in stopping cars but also in other ways: sandpapering wood, rubbing our hands together etc. The result is always the same - energy of motion is converted into work opposing friction and finally the work results in a heating of the surfaces (e.g., tire and roads).

We already have a feel for what the frictional force depends upon. First we know that friction decreases if we wet a surface (we skid farther). If the surface freezes then friction is reduced even more and we skid farther. Also we know that different surfaces affect the skid lengths and therefore the frictional force differently. For example, we know that a car going 30 mph will skid farther on loose gravel than on dry asphalt.

The second thing that affects the frictional force is an object's weight. More specifically the scientist would say the normal force. For our purposes we can use the weight; but we must realize that if the object is skidding down a hill the weight is less effective in producing slowing and if the object is sliding uphill the weight is more effective in producing slowing.

Let's start with horizontal road surfaces.



We have all heard of loading a trunk with bags of sand to improve traction (or friction) between the road surface and the tires. Also we all know that it is easier to push a VW on an icy road than to push a large Buick. Therefore, the larger the weight tie larger the frictional force. The formula we use is

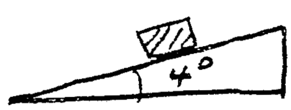
$$f = \mu w$$

Here $w = mg$ is the weight and μ is the coefficient of friction, represents the efficiency of producing friction when two surfaces are rubbed against one another, e.g., rubber tires on asphalt. The values of μ are as follows:

Typical Value Ranges		
	$0.6 \leftrightarrow 0.8$	rubber tires on dry asphalt
	$0.45 \leftrightarrow 0.7$	rubber tires on wet asphalt
	$0.1 \leftrightarrow 0.25$	rubber tires on ice

When a car skids to a stop work is done by the frictional force in stopping the car. It turns out that the average coefficient of friction for stopping a car decreases as the speed before skidding increases. This decrease occurs because the tires heat up when the skid begins and the rubber consequently breaks off more easily. The faster the speed before the skid, the hotter the tires get and therefore the lower the average coefficient of friction. Here we have introduced the average coefficient of friction because its value will change as the vehicle's speed decreases. Fortunately, we need only the average frictional force over the length of the skid and, therefore, only the average coefficient of friction over the length of the skid. All measured and tabulated values of μ for a car skidding to a stop are average values.

For level surfaces the coefficient of friction is equal to what is called a drag coefficient. On hills the so-called drag coefficient is different from the coefficient of friction because the weight is either more effective (uphill) or less effective (downhill) in stopping the skidding vehicle. For example for a 4° grade:



The uphill motion's drag coefficient is $\mu + 0.07$ and the down hill motion's drag coefficient will be $\mu - 0.07$ That is, for every degree one either adds or subtracts 0.0174 to the coefficient of friction for rubber tires on that surface.

The work done by (or against) the frictional force during a skid is given by:

$$W = f \times d$$

force = coefficient of friction × weight

$$f = \mu W = \mu m g \quad \{\text{for horizontal surfaces}\}$$

$$W = \mu m g d$$

coefficient of friction (μ); weight of vehicle in pounds (mg); skid distance in [feet] (d)

The total unit of work is given in ft.lbs.

$\frac{1}{2} v_1^2 = 2 \times \mu g d$

$2 \times .6 \times 32 \times 100 \quad - 38.40 = v_1^2$

f. Kinetic Energy - When a car (or anything for that matter) is in motion, we know that it has the potential to do work. For example, if a moving car hits and shears the trunk of the tree, we know that a force (exerted by the car) acted through a distance to shear off the tree. Since a force acted through a distance, work was done and we can represent it as $W = F \times d$ as we have previously discussed. We therefore have an intuitive feel for how work can be performed by moving vehicles. For example, we expec more damage in collisions (therefore more work done) for ehicles traveling at higher speeds. In fact, as the speed increases the potential for damage (and work) rapi y increases. Another factor which increases the dam e potential and therefore the work done is the weight ((more scientifically) the mass of the yourself as the owner of a car stalled in an intersection. You would expect more damage to be done to your beloved car if it were hit at 20 mph by a han by a Fiat. Furthermore, if your car must get hit you would prefer a bicycle at 20 mph to the Fiat. Therefore as the quantity of matter hitting your car is reduced so is the damage.

The scientific measure of how much work can be done by a moving vehicle (on level surfaces) is called its Kinetic Energy. Since the potential for damage and work increases as the speed and mass (or weight) we expect the Kinetic Energy to depend on both. It does and is given by the expression

$$KE = \frac{1}{2} m v^2$$

That is, $\frac{1}{2}$ times the mass, times the velocity squared.

g. Principle of Conservation of Energy - A fundamental precept of physics is that energy cannot be created nor destroyed. It can only be converted from one form to another. One can think of energy in terms of money.

friction = ($\mu \pm$ % grade) × weight   i.e. up hill or down hill

Weight = $m \times g$

If you have 3100 to spend you can record how every penney is spent. In subsequent transactions every portion of the initial $100 can be followed as the money changes hands several tines, The money is never destroyed, it is only traded from one person to the next. Energy is exchanged or rather changes form in a similar manner. Consider a car traveling zt a speed of 30 miles/hour. To be useful in tne Kinetic Energy formula we must first convert miles/hour to feet/sec as follows:

$$1 \text{ mile} = 5280 \text{ ft}$$

$$1 \text{ hr} = 60 \times 60 = 3600 \text{ sec}$$

$$\therefore \quad 1 \frac{\text{mile}}{\text{hour}} = \frac{5280 \text{ ft}}{3600 \text{ sec}}$$

$$\text{or} \quad 1 \frac{\text{mi}}{\text{hr}} = 1.467 \frac{\text{ft}}{\text{sec}}$$

$$\text{so} \quad 30 \frac{\text{mi}}{\text{hr}} = (1.467)(30) = 44 \frac{\text{ft}}{\text{sec}}$$

If the car weighs 3000 pounds then its mass can be computed as follows:

$$m = \frac{W}{g} = \frac{3000}{32} = 93.75 \text{ slugs}$$

Therefore:

$$KE = \frac{1}{2}(93.75)(44)^2$$

$$KE = 90{,}750 \text{ ft-lbs}$$

There are 90,750 ft-lbs of energy available **prior** to the skid. If the car skids to a stop all 90,750 ft.lbs are used to do work: against friction.

Work is defined as work = force x distance. If we consider F to be the force of friction we know that

$$F = \mu W$$

W ↳ weight

μ ↳ coefficient of friction

$$\therefore \quad \text{Work} = \mu w \times d$$

d ← distance

If all the KE is expended doing work against friction then

$$KE = \text{Work}$$

$$\text{or} \quad \tfrac{1}{2} m v^2 = \mu w \times d$$

For $v = 30 \text{ miles/hr} = 44 \text{ ft/sec}$ we have:

$$(\text{ft-lbs})\ 90{,}750 = \mu w \times d$$

This expression gives us a way to determine the coefficient of friction for the car which begins the skid at 30 miles/hour and skids to a stop. If the skid distance is 50 ft we can use it along with the weight (3000 lbs) in the formula. We obtain:

$$90{,}750 \text{ ft}\cdot\text{lbs} = \mu\,(3000 \text{ lbs})(50 \text{ ft})$$

$$\mu = 0.6$$

Values for the coefficient of friction have been tabulated for various kinds of road surfaces and for vehicles going at different speeds. So in most instances, μ can be considered as known in a given accident situation.

If one turns the equation around we see that it is possible to determine the speed before the skid started if the skid distance, weight and coefficient of friction are known.

Consider

$$KE = \text{Work due to Friction}$$

$$\tfrac{1}{2} m v^2 = F \times d$$

$$\tfrac{1}{2} m v^2 = \mu\, w \times d$$

if m, w, μ and d are known then the only unknown in the equation is v and it can be computed. This is exactly the situation encountered in many accidents involving skids. One must remember, however, that the above fornula only applies if to a stop. If it does not skid to a complete stop, then one must account for the energy which is not used to do work against the frictional force. We cuss this situation later on.

h. Momentum - As with the other physical quantities which we have defined, we also have a notion as to the scientific meaning of momentum. Momentum is similar to kinetic energy in that we associate more momentum with an object as it travels faster. Also we speak of increasing momentum when more and more people, things or particles are involved; like a wave in which the momentum builds up., the "momentum" turns in a basketball game, a political campaign builds up "momemtum". The physical definition of momentum is just the product of the mass of an object with its velocity

$$\text{Momentum} = mv$$

$m \times v$

If we are concerned about cars traveling on various surfaces then we can write the momentum in terms of the vehicle's weight. Recall

$$\text{weight} = mg$$

$$\text{where } g = 32\ ft/sec^2$$

$$\text{or } m = \frac{\text{weight}}{g}$$

Therefore:

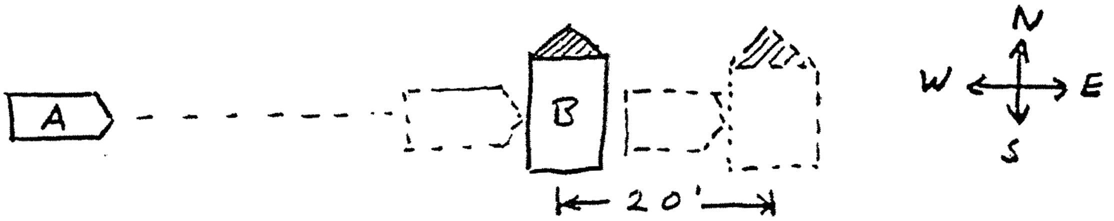
$$\text{Momentum} = \frac{\text{weight}}{g} \times v$$

Momentum is a special physical quantity in that it is a quantity which always has a direction associated with it. For example, a speed of 50mph when given a direction, e.g., southwest, becones a vector velocity. In order to use momentum effectively in computation one must know the direction of the momentum with respect to sone reference direction, e.g., the edge of a road. In car collisions we rarely have to worry, about changes in mass so the direction of a vehicles velocity is also the direction of its momentum. This particular momentum is called linear momentum. If a vehicle spins then the direction of the linear momentum which is important here is the direction the center of gravity travels. If a car spins during a skid, the spin will have no effect on the direction of the straight line skid and therefore no effect on the linear momentum direction of the skidding vehicle. This assumes that all four wheels are locked during the spinning skid.

The single most important physical Principle which is used in the analysis of auto collisions is the principle of "conservation of momentum." This principle connects the speeds (veiocitiea) of the vehicles after a collision with their counterparts before a collision, We will consider this principle next.

i. <u>Conservation of Momentum</u> - The principle of conservztion of momentum states that the total linear momentum of two vehicles <u>after</u> a collision is the same as it was <u>before</u> the collision. For collisions in which the initial and final velocities of the two vehicles are not along the same straight line one Is required to do vector addition. Vector addition is more tedious than complicated and we will save two-dimmensional collisions for later.

Let's consider the following collision in which all velocities before and after impact are along a single straight line.



Vehicle A (weight 3300 lbs) hits vehicle 3 (weight 4000 lbs). 3 has just pulled out of a driveway and is not moving in the east-west direction. A hits 3 going at speed V_A and then both A and 3 stick together and **skid** 20ft. to e stop.

Conservation of Momentum is invoked at impact.

Momentum Before = Momentum After

$$m_A V_A + m_B (0) = (m_A + m_B) V_{after}$$

B is stopped initially

Speed of the combination the instant after impact

This equation becomes

$$m_A V_A = (m_A + m_B) V_{After}$$

where $m_A = \frac{3000}{32}$ and $m_B = \frac{4000}{32}$

To get V_{after} we need to use the work-energy relation.

$$\frac{1}{2} (m_A + m_B) V_{after}^2 = \mu (m_A + m_B) g \times d$$

That is, the velocity of the combination immediately after impact is the same as their velocity at the instant the skid begins. Therefore, the same velocity appears in both the momentum and the energy equations. Suppose the coefficient of friction is $\mu = 0.7$

$$\frac{1}{2} V_{after}^2 = (0.7)(32)(20)$$

$$V_{after}^2 = 896$$

$$V_{after} = 29.9 \text{ ft/sec}$$

We can now gut this value of V_{after} into the conser-vztion of momentum equation to find the speed of A theinstant before the collision

$$m_A V_A = (m_A + m_B) V_{After}$$

$$\frac{3000}{32} V_A = \left(\frac{3000}{32} + \frac{4000}{32}\right)(29.9)$$

$$V_A = 69.8 \text{ ft/sec.}$$

$$V_A = \frac{69.8}{1.467} \frac{mi}{hr}$$

$$V_A = 47.6 \frac{mi}{hr}$$

Therefore, depending on the speed limit, visibility, obstacles (hill crest, etc.) one may be able to say whether or not the driver of vehicle A was or was not exceeding the speed limit. Notice that A's speed before the collision was more than twice its speed after coll-ision and that without the use of the principle of con-servation of momentum there would be no hope of even estimating it.

h. <u>Derivation of the Principle of Conservation of Momentum</u> - To understand why momentum is conserved we need only the second and third laws of old Isaac. His second law is

FORCE = MASS X ACCELERATION .

If we put in the definition of acceleration we get

$$F = m \frac{\Delta V}{t}$$

When we multiply both sides of this equation by the time t it becomes

$$Ft = m \Delta V$$

The left side is called the impulse and the right side is just the change in momentum of the mass m. Here the time t is the time period in which the force F acts to cause the change in momentum $m \Delta v$.

Recall that $\Delta V = V_{final} - V_{initial}$

Now the impulse equation becones:

$$Ft = m(V_{final} - V_{initial})$$

Let's consider tne momentum change of two cars due to an impulsive force acting on each for a given length of tine.

Car #1 $$F_1 t_1 = m_1 (V_{1\,final} - V_{1\,initial})$$

Car #2 $$F_2 t_2 = m_2 (V_{2\,final} - V_{2\,initial})$$

If the cars collide then Newton's third law says that

(Action force of 2 on 1) $F_1 = - F_2$ (Reaction force of 1 on 2)

In addition the time during which each car acts on the other (and therefore the time of sction for each force) is the same. It is just the tiae interval during which the two cars are in contact.

so $$t_1 = t_2 = t$$

and $$F_1 t_1 = -F_2 t_2$$

Now replacing the impulses by the respective changes in momenta we get:

$$m_1 (V_{1\,final} - V_{1\,initial}) = -m_2 (V_{2\,final} - V_{2\,initial})$$

or

$$m_1 V_{1\,final} - m_1 V_{1\,initial} = -m_2 V_{2\,final} + m_2 V_{2\,initial}$$

On rearranging terms this equation becomes:

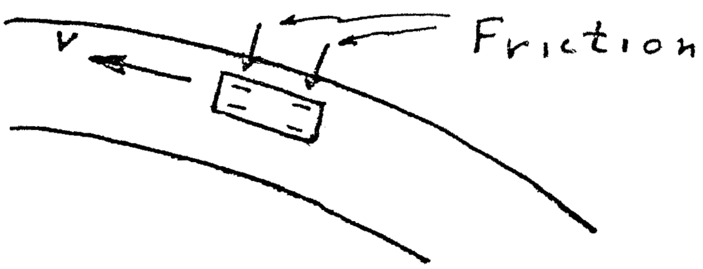
$$m_1 V_{1\,final} + m_2 V_{2\,final} = m_1 V_{1\,initial} + m_2 V_{2\,initial}$$

Total Momentum After the Collision = Total Momentum Before the Collision

Thus, we see that momentum is conserved in every collision.

i. <u>Centrifugal Force and Skidding on a Curve</u> - It is well know- that if one drives an automobile around a curve at a speed which is very high tne car will not hold the road and you feel as though you are skidiinp outward. Any object (including a car) which changes the direction of its velocity is accelerated. As a result, oy Newton's second law (F=ma) a force is needed to cause that acceleration. Consider a bill on a string. If you swing the ball in a circle it will be accelerated and the accelerating force is the tension in the string. If you cut the string the ball refuses to change the direction of its velocity and it will fly off as shown below.

For a car rounding a curve the force which holds the car on the curve and allows it to change the direction of its velocity is the frictional force between the edges of the tires and the road surface.



When we ride in a car which is rounding a curve we feel the centripetal acceleration and we experience the tendency to slide outward. Also we have all had our sunglasses or change or some other small items slide across the dashboard as we take a right turn rather rapidly. The frictional force for these small items and the dashboard is not great enough to cause their velocity to change direction. The acceleration which is termed centripetal acceleration of the car as it rounds a curve of radius R (given in feet) with a speed v (given in feet /sec) is:

$$a = \frac{v^2}{R}$$

Therefore the centifugal force is just this acceleration times the mass.

$$F = m\frac{V^2}{R}$$

The maximum force that friction can exert on the car to keep it on the curved path. is just the same as the frictional force which we used in the analysis of straight line skids. Only here the frictional force is scting in a different direction. It is:

$$F = \mu m g$$

When $\frac{mV^2}{R}$ is greater than $\mu m g$ the frictional forces are not strong enough to change the car's direction and keep it traveling on the curved path. is a result, when

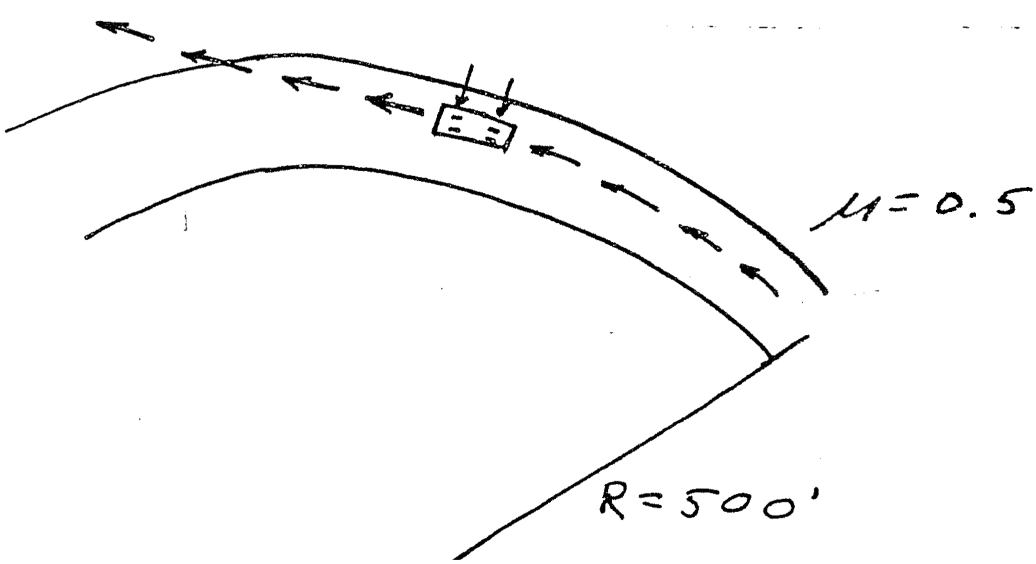
$$m\frac{V^2}{R} = \mu m g$$

then for a given R and μ we can compute the maximum speed which the car may hzve and hold its curved path. Looked at another way this speed is zlso the minimum speed which will cause the vehicle to slide out of its initially curved path. For example:

Let $R = 500 ft$
$\mu = 0.5$

$$\cancel{m}\frac{V^2}{R} = \mu \cancel{m} g$$
$$V^2 = \mu g R$$
$$V^2 = (.5)(32)(500)$$
$$V^2 = 8000$$
$$V = 89.44\ ft/sec = 60.58\ mi/hr$$

So if the car exceeds 60.58 mi/hr on this curve it will not hold the curve and it will go off the road following ~;-",epath shown below.



Evidentiary Analysis of Traffic Accidents

I. Introduction

a) Assumptions, Objectives, and Limitations of Accident Reconstruction

b) The Scientific Method as applied to Traffic Accident Reconstruction

c) Human Factors

II. Principles

a) Ehysical Terms and Definitions

b) The Laws of Motion

c) The Laws of Conservation of Momentum and Energy

d) Review of Simple Algebra

III. Stopping and Skidding

a) Skidmarks

b) Frictional Force and Work

c) Coefficient of Friction--definition and values for different surfaces

d) Speed Computations from Skidmarks

IV. Collisions

a) Line of Impact and Point of Impact

b) One Dimensional Collision Analysis

c) Two Dimensional Collision Analysis

d) Problems Involved in Estimating Speeds from Auto Damage

V. Time and Position--their special importance in traffic zccident reconstruction

VI, Reconstruction of Actual Traffic Accidents from Police Reports and Testimony

Each example will be viewed with special attention to:

a) The accident data
b) Required assumptions
c) Reconstruction results

VII. "Expert" Testimony

Books

"Traffic Accident Investigation Manual"
by J. Stannard Baker (The Traffic Institute, Northwestern University) 1975

"Highway Collision Analysis" by J. C. Collins and J. L. Morris; Charles C. Thomas Publishing (Springfield, Ill., 1967)

"Estimating Stopping Distance and Time for Motor Vehicles" by J. Stannard Baker (The Traffic Institute, Northwestern University) 1977

"Forensic Physics" by David L. Uhrich (Kent State Printing Service, 1976)

David L. Unrich

Materials whicn are useful in Accident Reconstruction

1. The Police Report,
2. Pictures of the Accident Scene.
3. Pictures of the Accident Vehicles.
4. Copies of Statements or Depositions made by the participants and witnesses of the accident.
5. A copy of the Engineering Drawing of the road or Intersection where the accident occurred.
6. The Weights of the vehicles involved in the accident or of comparable vehicles of the same make, model and year,,

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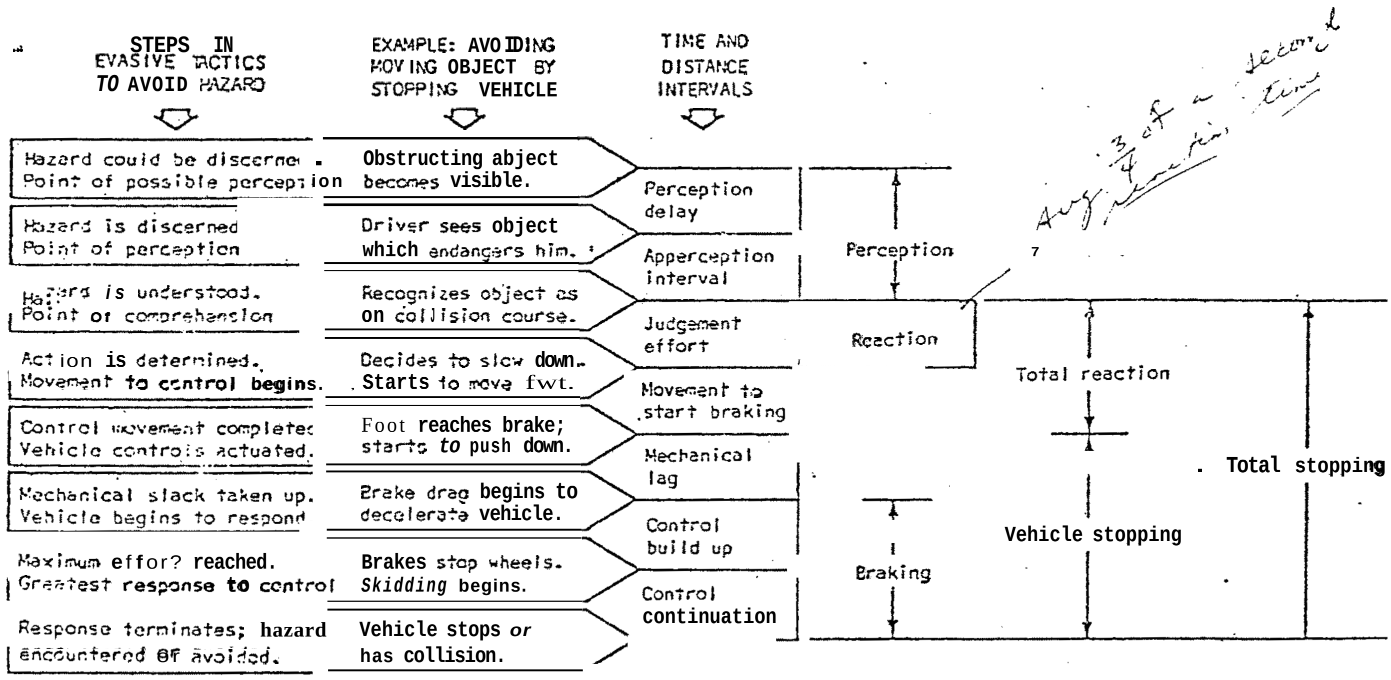
DESCRIPTION OF ROAD SURFACE	DRY				WET			
	Less thao 30 mph		More than 30 mph		Less than 30 mph		More than 30 mph	
	From	To	From	To	From	To	From	To
PORTLAND CEMENT								
New, Sharp	.80	1.20	.70	1.00	.50	.80	.40	.75
Travelled	.60	.80	.60	.75	.45	.70	.45	.65
Traffic Polished	.55	.75	.50	.65	.45	.65	.45	.60
ASPHALT or TAR								
New, Sharp	.80	1.20	.65	1.00	.50	.80	.45	.75
Travelled	.60	.80	.55	.70	.45	.70	.40	.65
Traffic Polished	.55	.75	.45	.65	.45	.65	.40	.60
Excess Tar	.50	.60	.35	.60	.30	.60	.25	.55
GRAVEL								
Packed, Oiled	.55	.85	.50	.80	.40	.80	.40	.60
Loose	.40	.70	.40	.70	.45	.75	.45	.75
CINDERS								
Packed	.50	.70	.50	.70	.65	.75	.65	.75
ROCK								
Crushed	.55	.75	.55	.75	.55	.75	.55	.75
ICE								
Smooth	.10	.25	.07	.20	.05	.10	.05	.10
SNOW								
Packed	.30	.55	.35	.55	.30	.60	.30	.60
Loose	.10	.25	.10	.20	.30	.60	.30	.60

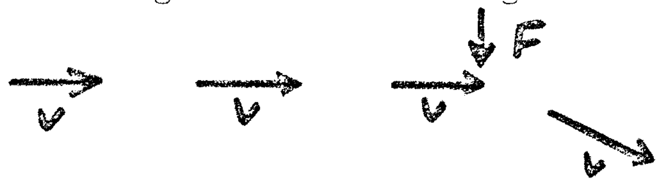
The lower the coefficient, the greater the stopping distance required

Exhibit 1-1

EVENTS DURING EMERGENCY BRAKING

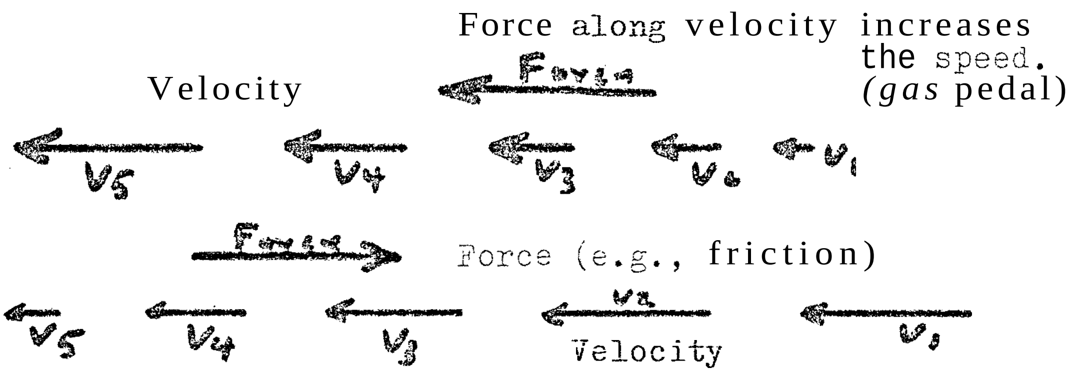


Centrifugal Force--Skidding on a Curve



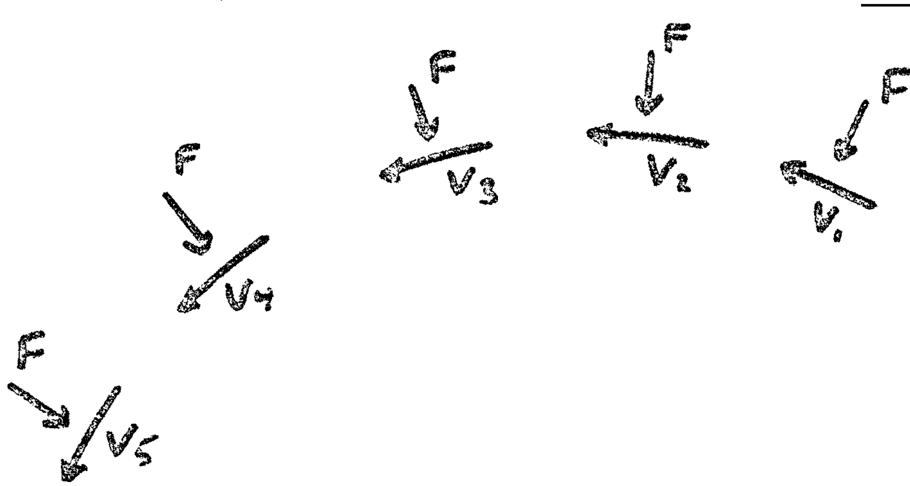
To cnange the direction of motion a Force must be exerted perpendicular to the line of motion.

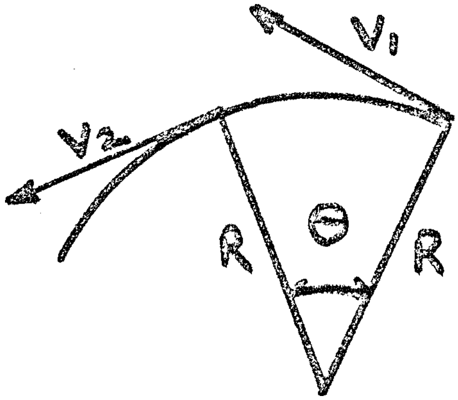
Since the force is perpendicular tc the direction of v it can neither slow nor increase the speed.

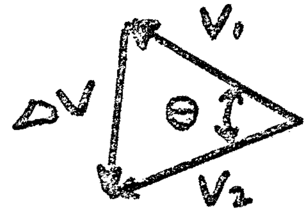


Force anti-parallel to velocity then speed decreases. (brake)

Force ⊥ to v changes the direction of v <u>only</u>.

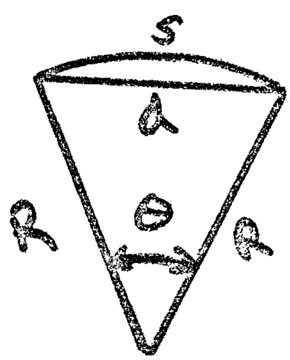






$V_2 = V_1 = V$

$$a_{centrifugal} = \frac{\Delta V}{t}$$



$d \approx s$

$d = R\theta$

or $\theta = d/R$

$\Delta V = V\theta$

or $\theta = \frac{\Delta V}{V}$ so $\frac{\Delta V}{V} =$

But $a_{centrifugal} = \frac{\Delta V}{t} = \frac{d}{t}\,\frac{V}{R}$

but $d/t = V$

So $a_c = \frac{V \cdot V}{R} = \frac{V^2}{R}$

Force needed to keep object (car) moving in a curved path is ⊥ to direction of motion and

$$F_c = m\frac{v^2}{R}$$

For car on a road the force is friction of the road surface acting on the rubber tires.

Maximum v for which car will not slide out of the curve is

$$\frac{m V^2_{max}}{R} = \underbrace{\mu m g}_{\text{Frictional Force}}$$

The mass cancels out:

$$\therefore \quad \frac{V^2_{max}}{R} = \mu g$$

or

$$V^2_{max} = \mu g R$$

Let $\mu = 0.5$; $R = 500'$; $g = 32.2\ ft/sec^2$

$$V^2 = (.5)(32)(500)$$

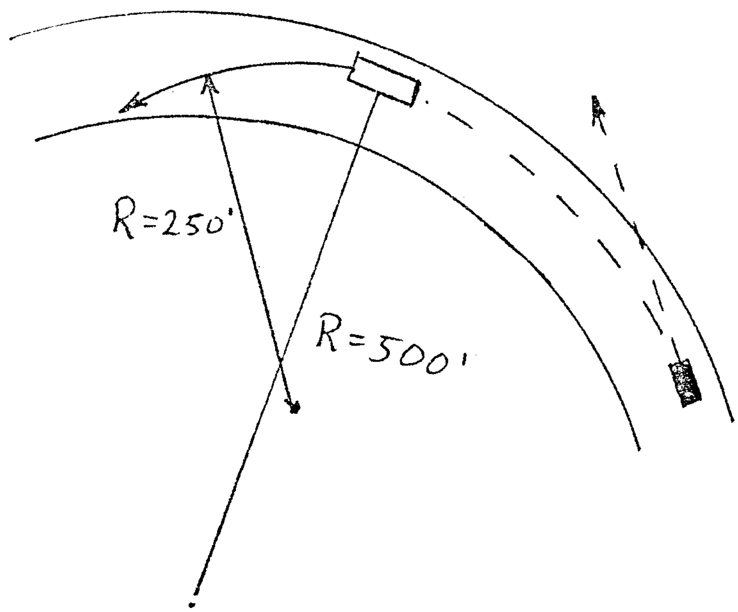
$$V^2 = 8000$$

$$V = 89.4\ ft/sec$$

$$V = 89.4\left(60/88\right) \approx 61\ mi/hr$$

If v > 61 mi/hr, car will not be able to hold the curve.

Note that if when you. realize you are not holding the curve, you turn the wheel sharper--you make R smaller!!



Say car curve is 250 ft

$$V^2 = (0.5)(32)(250)$$

$$V^2 = 4000$$

$$V = 63.2 \text{ ft/sec}$$

$$V = 63.2\,(60/88) = 43 \text{ mph}$$

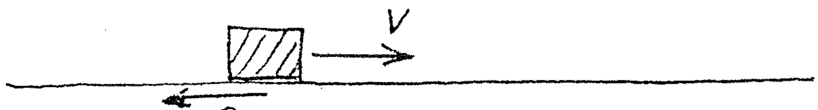
Maximum sped for which one will be able to negociate the curve safely is reduced from

61 tc 43 mi/hr.

If the curve is banked, then the coefficient of friction is augmented by adding or subtracting the % grade.

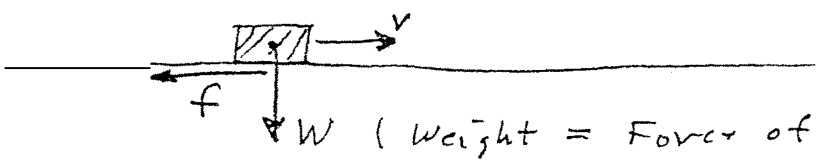
I Uhrich

Consider an Horizontal Surface



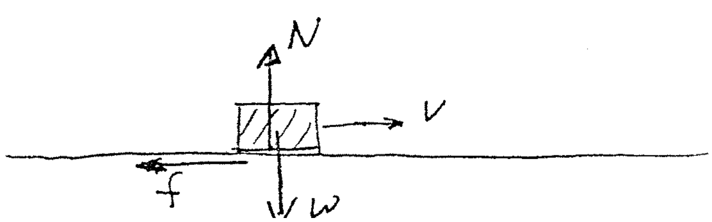
f ← frictional force opposes the motion

Let's add gravity to the problem



W (Weight = Force of gravity pulling down)

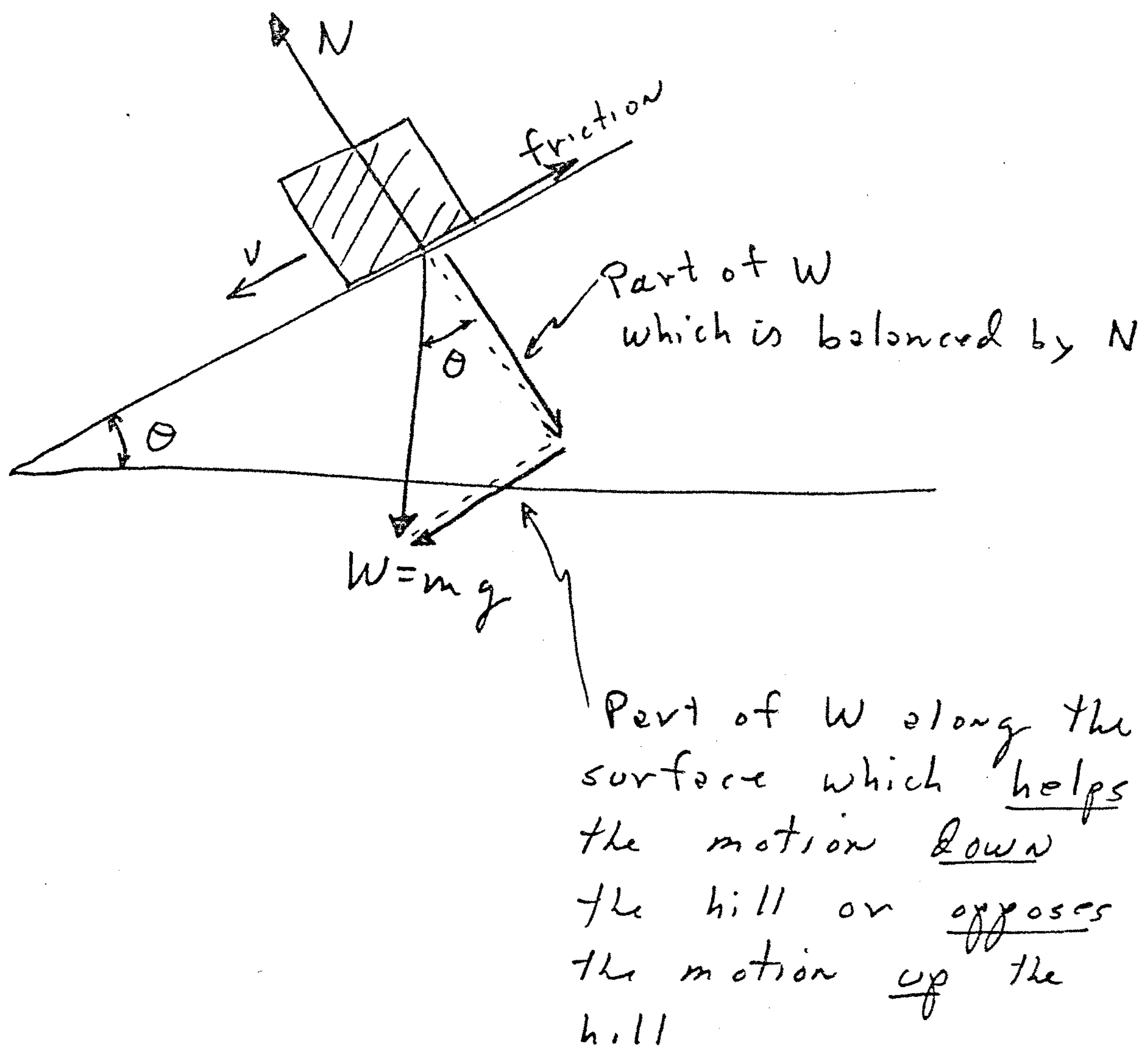
Object is in vertical Equilibrium because surface pushes up to balance the gravitational force.



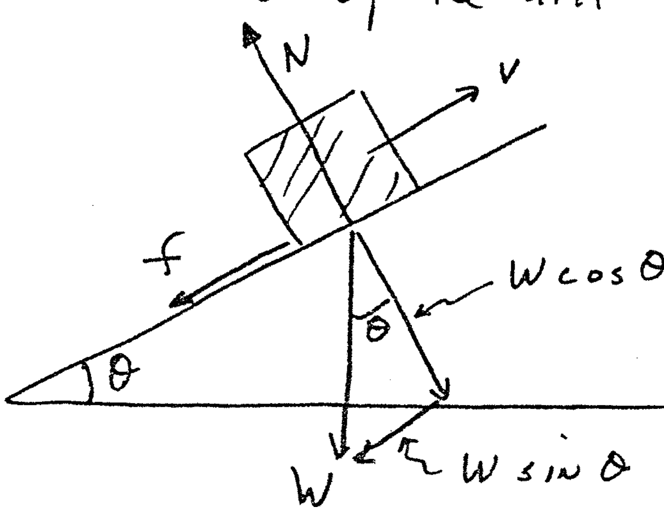
The force of the surface (up) is called the normal force (N)

Since there is vertical equilibrium

$$W = N$$



Consider motion up the hill



$N = W\cos\theta$

$\therefore$ frictional force $= \mu N = \mu W \cos\theta$

Therefore, for angles even up to 20° the cos is on the order of 1. Therefore.

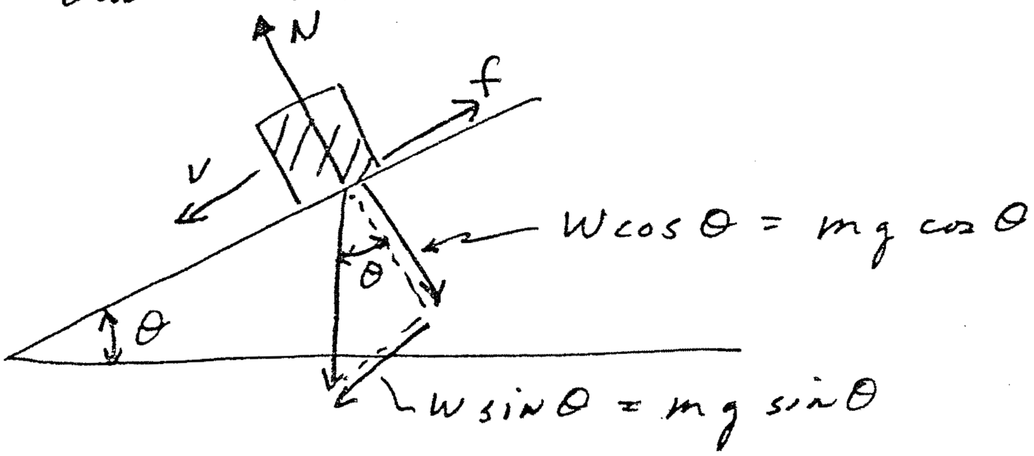
$$F_{\text{total opposing motion}} = mg(\mu \cos\theta + \sin\theta)$$

(with $\cos\theta \to 1$)

Therefore $F_{total} = mg(\mu + \sin\theta)$

where $\sin\theta \approx 0.0174$ for each degree (1° of grade)

For downhill motion



Now friction (f) and $mg\sin\theta$ are in opposite directions. Thus the total force is given by

$$F_{total} \quad mg(\mu \cos\theta \qquad \theta)$$

NB This is not in Traffic manual book for high grade calculations

plus sign for an uphill slide.

The total force becomes

$$F_{tot} = mg\,(\mu \cos\theta \pm \%\ \text{grade})$$

as a decimal (% grade)

minus sign for a downhill slide

$(\mu \pm \%\ \text{grade})$ = drag factor for a hill or grade.

<u>Agonizing Reappraisal</u>

Fundamental Quantities

1. length (ft, mi)
2. mass (slugs)
3. time (sec, hr)

Derived quantities

1. velocity (ft/sec, mi/hr) $v = d/t$
   (recall 88 ft/sec = GO mi/hr)
2. acceleration (ft/sec^2) $a = v/t$
3. Eorce (lbs) $F = ma$

   e.g., 1. weight $w = mg$

   where $g = 32.2\ ft/sec^2$

   2. friction $f = \mu mg$

   on a horizontal surface and all the weight is useful in slowing a vehicle

   on a hill $f = mg(\mu \pm \%\ grade)$ (+ uphill, − downhill)

4. Work = Force x distance (ft · lbs)

   They must be along the same direction

   $W = F \times d$
   for friction $f = \mu mg$

   So Work $= \mu mgd$

5. Kinetic Energy (Energy of Motion)

   $K.E. = \frac{1}{2}mv^2$ (ft · lbs)

6. Conservation of Energy

   e.g., Motional Energy is used up (as a car skids to at stop) doing work against the frictional force.

   KE = Work against friction

$\frac{1}{2}mv^2 = \mu mgd$

Note mass cancels out.

so $\frac{v^2}{2} = \mu gd$

Example: Find v if $\mu = .7$ and $d = 50$ ft.

$v^2 = 2\mu gd$

$v2 = 2(.7)(32.2)(50)$

$v^2 = 2254$

$v = \sqrt{2254} = 47.5$ ft/sec

$v = 47.5(\frac{88}{88}) = 32.4$ mi/hr

If the vehicle weighed 3000 lbs, then how much K.E. did it have?

$KE = \frac{1}{2}mv^2$

$= \frac{1}{2}(\frac{3000}{32.2})(47.5)^2$

$KE = 105{,}105$ ft·lbs

What happens if the car does not skid to a stop?

Then $KE_{beginning} - W_{work\ in\ skid}$ I- $KE_{end\ of\ skid}$

V_o = beginning speed

V_f = speed at end of skid

$\frac{1}{2}mV_o^2 = \mu mgd + \frac{1}{2}mV_f^2$ (mass cancels out)

Let $V_f = 15$ mph

$V_f = 15(\frac{88}{60}) = 22$ ft/sec

$\frac{V_o^2}{2} = \mu gd$ I- $\frac{V_f^2}{2}$

$V_o^2 = 2\mu gd + V_f^2$

$V_o^2 = 2(.7)(32.2)(50)$ I- $(22)^2$

$V_o^2 = 2254 + 484$

$V_o^2 = 2738$

$V_o = \sqrt{2738} = 52.3$ ft/sec

$V_o = 52.3(\frac{60}{88}) = 35.7$ mi/

Notice 15 mi/hr at the end of the 50 ft skid adds only 3.9 mi/hr to the speed at the beginning of the skid..

Note: If only two of the four wheels are effective in slowing the vehicle--then only the weight on these wheels contributes to the slowing of the vehicle.

e.g. $\mu = .7$

$d = 50$ ft

and the skid is to a stop

$\frac{1}{2}mv^2 = \mu(\frac{mg}{2})d$

$v^2 = (.7)(32.2)(50)$

$v^2 = 1127$

$v = \sqrt{1127} = 33.6$ ft/se

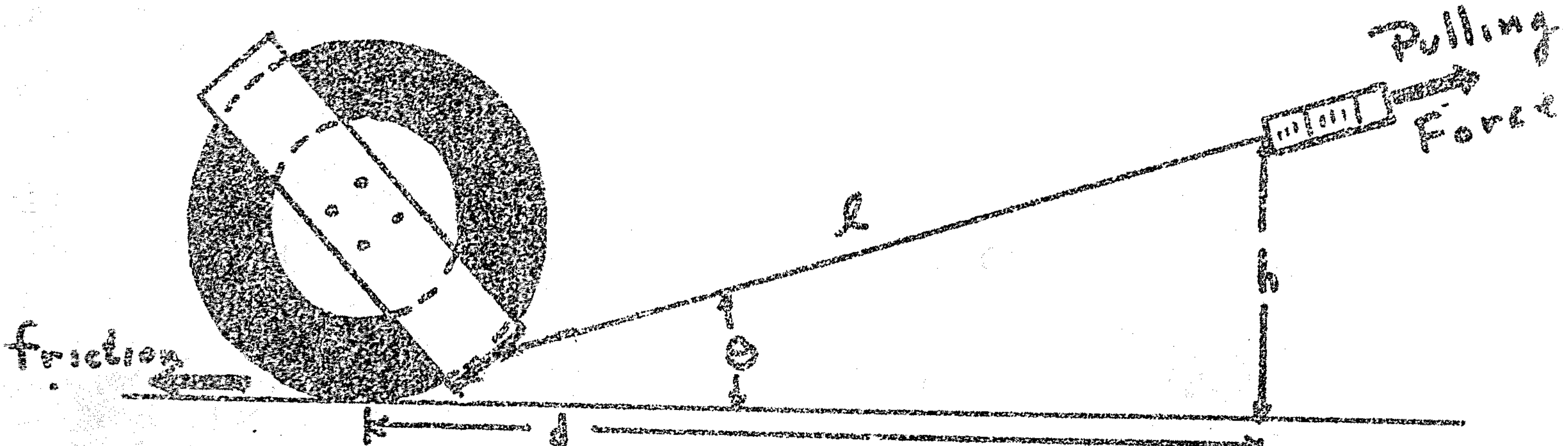
$= 33.6(\frac{60}{88}) = 22.9$ mi/hr

So if only $\frac{1}{2}$ of the weight is useful in slowing the car, tne speed at the start of the 50 ft skid. is reduced from 32.4 to 22.9 mi/hr.

Direct Measurement of
the Coefficient of Friction

David L. Uhrich

In order to eliminate the numerous errors involved in the measurement of the coefficient of friction with bicycles, we will use the following apparatus:



Here: ℓ = the distance from where the tire touches the pavement along the pulling rope to the spring balance hook

h = the vertical height of the end of the pulling rope above the ground

d = horizontal distance from the tire to the end of the pulling rope

θ = angle the pulling rope makes with the ground

When the tire is dragged at constant speed over a surface (asphalt, grass, concrete, etc.), the acceleration is necessarily zero.

$$a = \frac{V_{final} - V_{initial}}{t} = 0$$

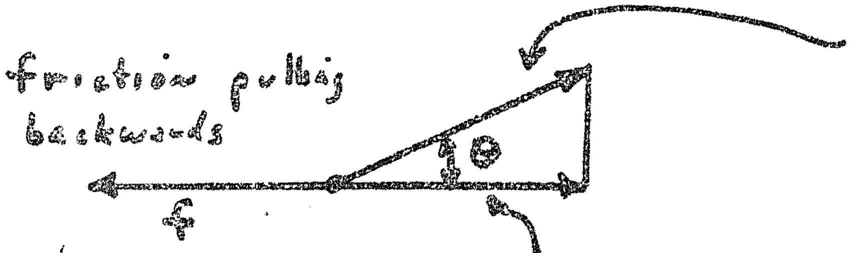
If the acceleration is zero, Newton tells us that the net force (horizontally in this case) is also zero.

$$F_{net\,(horizontal)} = ma = 0$$

But the net force here is just friction pulling left minus the horizontal component of the rope's force pulling right.

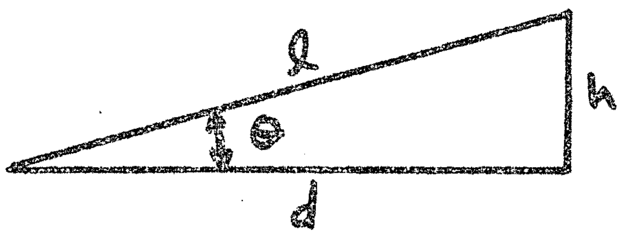
$$F_{net\,(horizontal)} = F_{Rope\ Horizontal} - f_{friction} = 0$$

$$\text{So} \quad f = F_{Rope\ Horizontal}$$



Rope pulling right at an angle θ above the horizontal. (F_{Rope})

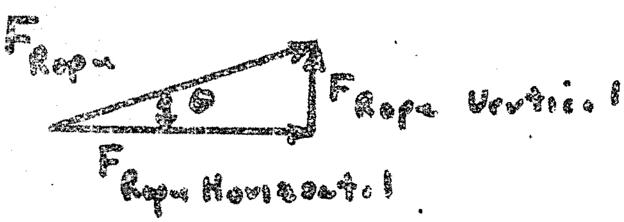
Horizontal component of the rope's force which just balances the pull in the opposite direction due to friction. ($F_{Rope\ Horizontal}$)



$$\sin\theta = \frac{h}{l}$$

$$\therefore \text{Inverse } \sin\theta = \theta$$

On the calculator divide h by ℓ then press <u>inverse</u> and then <u>sin</u> and the calculator will present you with the angle θ.



$$\sin\theta = \frac{F_{Rope\ Vertical}}{F_{Rope}}$$

$$\cos\theta = \frac{F_{Rope\ Horizontal}}{F_{Rope}}$$

F_{Rope} - That is, the total force with which you pull along the rope is just the measured reading on the spring scale.

Once you know θ, you can just enter θ into the calculator and then press the cosine button and you will have a numerical value for the cos θ.

Then if we multiply the above equation for cos θ by F_{Rope} we get:

$$F_{Rope} \cos\theta = \frac{F_{Rope\ Horizontal}}{\cancel{F_{Rope}}} \times \cancel{F_{Rope}}$$

So: $F_{Rope} \cos\theta = F_{Rope\ Horizontal}$

But $f = F_{Rope\ Horizontal}$ (f ← friction) $\left\{ \text{Also } F_{Rope\ Vertical} = F_{Rope} \sin\theta \right\}$

So: $f = F_{Rope} \cos\theta$

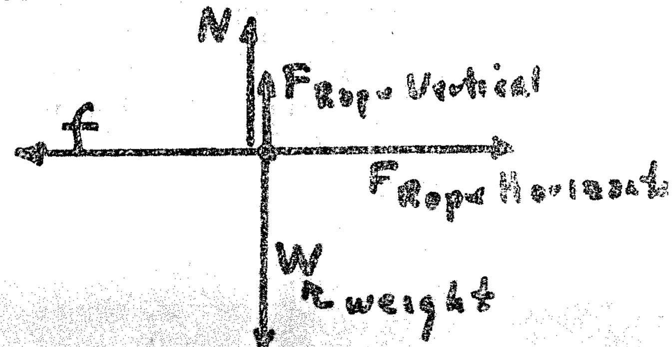
But the frictional force for horizontal surfaces is just the coefficient of friction times the normal force. The normal force is the vertical force which the surface exerts on the tire.

$$f = \mu N$$

(μ ← coefficient of friction; N ← Normal Force)

6

Here, there are three forces which are along the vertical or have a component in the vertical direction; the normal force of the surface on the tire (up), the weight or alteration of the earth for the tire (down) and the vertical component of the rope's force (up). Since there is no acceleration in the vertical direction, we have an equilibrium situation described again by Newton's first law.



$$F_{net\ (vertical)} = ma = 0$$

$$F_{net\ (vertical)} = N + F_{Rope\ Vertical} - W = 0$$

So: $N = W - F_{Rope\ Vertical}$

and $f = \mu N = \mu (W - F_{Rope\ Vertical})$

But from the horizontal equilibrium condition, we also have $f = F_{Rope} \cos\theta$.

Equating the two expressions for f gives

$$\mu (W - F_{Rope} \sin\theta) = F_{Rope} \cos\theta$$

Dividing each side by $(W - F_{Rope} \sin\theta)$ yields

$$\mu = (F_{Rope} \cos\theta)/(W - F_{Rope} \cos\theta)$$

Here you measure the pull on the rope in pounds (F_{Rope}); you can weigh the tire and frame with your spring scale just by lifting the frame with the hook on the end of the scale (so you measure W in pounds) and you also need to measure h and ℓ to determine the sin θ and cos θ. If every time you pull the tire you hold the rope to the same height h, you need to measure h and ℓ only once. Measure μ for asphalt, concrete, grass and other surfaces. For example, you could compare new asphalt with worn asphalt or try asphalt with dirt spread on it. Also determine if the pulling force depends on how fast you pull the tire. (It shouldn't!)

Each group should measure the coefficient of friction for <u>each</u> surface <u>with</u> <u>each</u> tire and determine if the measured values depend on the tire used. (e.g., does μ depend on the presence or absence of trend).

In your report draw conclusions from your own data regarding the nature of the coefficient of friction between sliding rubber tires on different pavements and compare this method to the use of skidding bicycles. Also comment on the fact that in an accident skidmarks indicate a skid but here you measure a coefficient of friction and you leave no skidmarks. Are the μ's the same?